How much is in there? A Polygon Investigation

I. ASSESSMENT TASK OVERVIEW & PURPOSE:

The student will use prior knowledge to investigate and create a method for finding the sum of the interior angles of any polygon. This is an investigative activity that will be used to draw the student into the unit and help the student personalize and internalize the new information. The student will create a presentation of their investigation and method at the end of the activity.

II. UNIT AUTHOR:

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III. COURSE:

Math 6, Math 7, Math 8, Algebra 1 or Geometry

IV. CONTENT STRAND:

Geometry, Algebra, Data Analysis

V. OBJECTIVES:

The learner will be able to create, use, and explain a formula for determining the number of degrees inside any polygon.

VI. REFERENCE/RESOURCE MATERIALS:

The students will need access to paper, poster board, markers, pipe cleaners, craft sticks, toothpicks, straws, graph paper, construction paper, protractor, string and any other manipulative materials the teacher may have to assist the investigation and presentation. The students may use computer software (GeoGebra, Geometer's Sketchpad, PowerPoint, Prezi, etc.) to create and present their investigation.

VII. PRIMARY ASSESSMENT STRATEGIES:

The students' final presentation will be evaluated on presentation and explanation of the students' investigation into the sum of the interior angles of a polygon. The students will be evaluated on explanation of method and visual representation of investigation. The attached rubric for the project and their participation in the process will be provided to the students.

VIII. EVALUATION CRITERIA:

The rubrics at the end of this document will be used by the student to self-evaluate, evaluate group members, and by the teacher for final evaluation.

IX. INSTRUCTIONAL TIME:

This activity is intended to take two or three 70 minute class periods, two 90 minute class periods or four 45 minute class periods.

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Strand

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Mathematical Goals and Objective(s)

The student will use prior knowledge to investigate and create a method for finding the sum of the interior angles of any polygon. The student will create a presentation of their investigation and method at the end of the activity.

Related SOL – *This lesson can be adapted/used for any of these levels but is typically a Geometry lesson*

- *SOL 6.17:* The student will identify and extend geometric or arithmetic sequences.
- *SOL 7.17:* The student will represent relationships with tables, graphs, rules and words.
- *SOL 8.14:* The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.
- *SOL A.1:* The student will represent verbal quantitative situations algebraically and evaluate the expression for given replacement values of the variable.
- *SOL G.10:* The student will solve real world problems involving angles of a polygon.
- *SOL AII.2*: The student will investigate and apply the properties of geometric sequences and series to solve real world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.

NCTM Standards

Algebra

- Understand patterns, relationships, and functions
  - Generalize patterns using explicitly defined and recursively defined functions.
  - Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes and local and global behavior.
- Represent and analyze mathematical situations and structures using algebraic symbols
  - Use symbolic algebra to represent and explain mathematical relationships.
  - Use mathematical models to represent and understand quantitative relationships.
- Use symbolic expressions, including iterative and recursive formulas, to represent relationships arising from various contexts.
  - Draw reasonable conclusions about a situation being modeled.

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
  - Analyze properties and determine attributes of two- and three-dimensional objects.
  - Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.
  - Draw geometric objects with specific properties, such as side lengths and angle measures.
  - Use geometric models to represent and explain numerical and algebraic relationships.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Make and investigate mathematical conjectures.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas build on one another to produce a coherent whole.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

Materials/Resources

- Pipe cleaners
- Craft sticks
- Toothpicks
- Straws
- Graph paper
- Construction paper
- Protractor
- String
- Poster board or large paper for presentations
- Geometry software for creating models and presentations (GeoGebra, Geometer's Sketchpad, PowerPoint, Prezi, etc.)

Assumption of Prior Knowledge

- The student should have an understanding of the definitions of line segments and measurement of angles. The student should also have an understanding of the definitions of triangles and understand that the sum of the interior angles of a triangle is equal to 180 degrees. The student should also understand that a straight angle measures 180 degrees and a circle measures 360 degrees. A basic knowledge of different types of polygons based on the number of sides will be helpful.
- Students should be operating on Analysis level 2 on the Van Hiele scale with respect to angle measurement and polygons.
- Students may have difficulty relating the number of sides to the sum of the interior angles. The teacher may need to prompt the students into thinking about the problem using all characteristics of the polygon as well as the prior knowledge stated above.

- Prior to this activity, the students will have been introduced to triangles and quadrilaterals. This includes definitions and discussion of the sum of interior and exterior angles for each figure.
- Students will be asked to consider the use of polygons in landscape design, architecture and interior design as an extension of the initial investigation.
- Students are familiar with giving and receiving Warm Feedback and Cool Feedback as a part of a Gallery Walk. Feedback during a Gallery Walk is anonymous and is used to edit and polish the group's final presentation. Examples of warm feedback include " I see excellent use of triangles to divide the polygons", "Attention to detail is evident in the following areas…" "I like the way you…". Examples of cool feedback include "I wonder if this fits that.", "It looks like you wanted this to do that, but I'm not sure it does.", "I can't tell if the student is to do this or that.", "It is hard to tell what the goal was, and what the thinking skills were."

Introduction: Setting Up the Mathematical Task

Task:

"A classmate has been absent through the polygons unit and you have volunteered to tutor your classmate to prepare them for the unit test. Your classmate has informed you that he learns better with hands-on activities. Using materials you find in your craft closet, you are to model how you and your classmate would create a hands-on method for finding the sum of the interior angles of any polygon and develop a mathematical formula that results from your investigation. Your presentation must include your models, a table of the data you gathered, and your method for find the sum of the interior angles of any polygons."

Timeline:

- **Class period 1:** Introduce the task to the class. Give the students 10 minutes to brainstorm at their own desks.

  Questions might include:

  - What type of polygon(s) will you build?
  - What materials will you need?
  - How many different types of polygons should you use to test your method?

  Organize the students into duos or triads for the remainder of the class period. The teacher should make their way from group to group to listen to the discussion and possibly offer prompts or questions to encourage student thinking.

  Questions might include:

  - How would you describe the problem in your own words?
  - What do you know that is not stated in the problem?
  - What if you could only use one polygon to create all the other polygons?
  - What assumptions are you making?

  Students need to complete creation of models and begin organizing data in a table for analysis. Students will have time in the next class period to continue with modeling and analysis.
- **Class period 2:** Students return to groups to finish modeling and data gathering. By the end of the period, groups should have created a rough draft presentation of their models with a display of their data and an explanation of their method for determining the sum of

interior angles of any polygon. Student homework for this day is a reflective journal entry on what they have learned so far and how the student feels the presentation is progressing.

- **Class period 3:** Students will display their presentations for their classmates. The students will complete a Gallery Walk of each other's work and leave warm and cool feedback (20 minutes). At the conclusion of the Gallery Walk, groups will have 30 minutes to refine their presentations. Students will present their final draft of their research and methods to the class at the end of the editing period.

Lesson Starters

- Model a series of polygons and measure the interior angles of each with a protractor.
- Model a series of polygons and divide them into triangles.
- Brainstorm to find the degrees in the polygon with the fewest sides before modeling.
- Start with a triangle and add triangles to create new polygons.

Instructional Techniques

- Brainstorming
- Pair students to move from brainstorming to models to data to formulas.
- Gallery walk to view other students work and leave questions and feedback.
- Use feedback to fine tune presentation.
- Reflective journal entry.
- Students will evaluate each other on their participation in the investigation and presentation.

Student Exploration

The order and components of exploration is up to you. The following are recommended as a part of this investigation

Individual Work - Brainstorming
Small Group Work - Models, Data table and Presentation
Whole Class Sharing/Discussion - Gallery Walk with Feedback

Student/Teacher Actions:

- During this exploration, the teacher presents the initial situation and monitors student progress and feedback.
- The teacher should be monitoring group discussions and directing student discussion. through misconceptions and roadblocks to understanding through leading questions such as:
  - What do you need to find out?
  - What information do you have?
  - Would it help to create a table?
  - What concepts that we have learned before could be useful in solving this problem?
  - What do you know that is not stated in the problem?
  - How would you describe the problem in your own words?
  - What do you know that is not stated in the problem?
  - Can you describe your method to us?
  - What if you could only use one polygon to create all the other polygons?
  - What assumptions are you making?
- During this exploration, the student is responsible for creating and analyzing models of various polygons to determine a method for finding the sum of the interior angles of any polygon. The student is expected to actively and appropriately participate in his/her group's discussion and creation of the group presentation. The student is also expected to

leave appropriate feedback during the Gallery Walk and help his/her group edit the final presentation to reflect feedback that has been received.
- Include ideas for technology integration or cooperative/collaborative learning within the student/teacher actions.

Monitoring Student Responses

- Students should be able to describe their method in words and as a formula.
- Students should be able to present evidence of their investigation through models and data tables.
- Students should be able to verbalize brainstorming ideas to his/her group and respectfully participate in group discussion as a part of the creation of the final presentation.
- Students should be able to provide warm and cool feedback to other groups as a part of the Gallery Walk activity.
- Students should be able to work with group members to edit and polish presentation after receiving feedback.
- Students will be asked to assess each other on their participation in the investigation and presentation.
- Teacher and students should be able to discuss areas of confusion or a misconception as the investigation is being completed.
- Teacher should be actively listening to groups as they investigate and create presentations to address issues of misconceptions or difficulty
- Teacher should ask groups probing questions (see above) to help students resolve roadblocks or misconceptions
- Teacher should include extension activity for students who finish early (Investigate sum of exterior angles based on your investigation of interior angles)
- At the end of each class period, the teacher will ask each group to report on their progress to the entire class. On the final day of the investigation, each group will present and discuss the results of their investigation.

Assessment List and Benchmarks

- Benchmarks:
  - Brainstorm List
  - Models of polygons divided into other polygons
  - Table of data from models
  - Explanation of method for finding sum of interior angles of a polygon
  - General formula for finding sum of interior angles of a polygon
  - Rough Draft of Presentation for Gallery Walk
- **Journal/writing prompts**
  - What was the hardest part of today's task?
  - What was the easiest?
  - What NEW information did you learn?
- **Other**
  - Gallery Walk: Each group will post its solution and classmates will look at each solution and leave one warm and one cool comment on a Post It note with no talking between students. After the Walk is finished, students will have 2 minutes each to reply.

- After posting all solutions at the front of the room, ask students to identify similarities and differences. Ask the class to come up with a common formula based on all the solutions presented.

- The activity can be adapted for students without prior knowledge by creating two to three models of the divided polygons as a class after the brainstorming time and before the students are sent to work in their groups.
- Possible Extensions: An extension idea worth exploring is that, as the number of sides in a regular polygon increases, the more that polygon beings to look like a circle. See these explorations for ideas:
  - http://www.mathopenref.com/polygonregular.html
  - https://tube.geogebra.org/m/DYJt4Rk6

Rubric for student self-evaluation and teacher evaluation of final presentation (teacher evaluation will be grade recorded in gradebook):

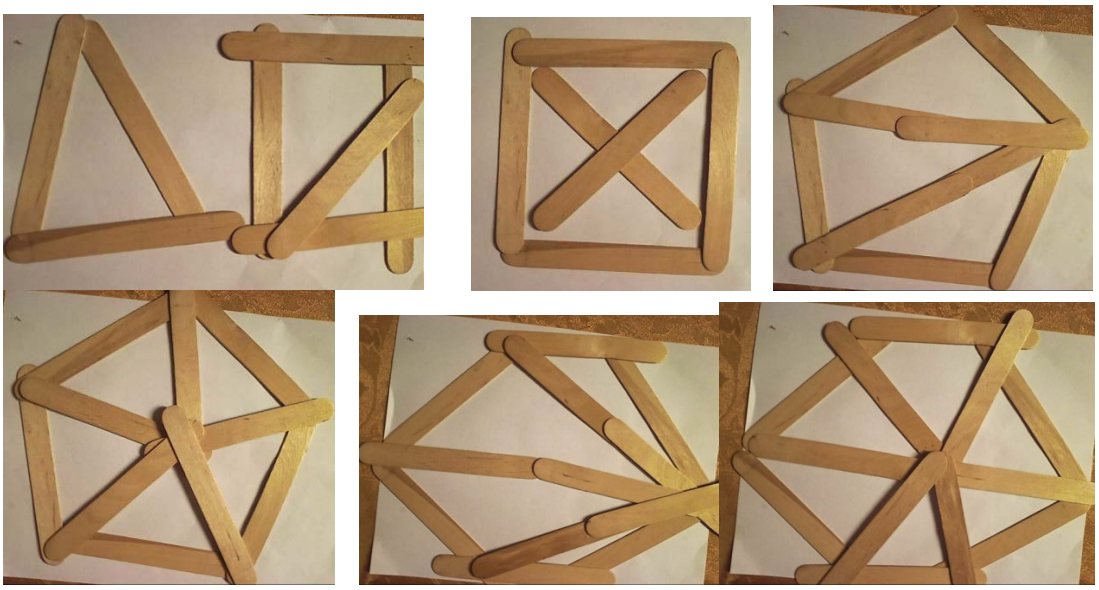
	Below Standard (5 pts)	**Almost Meets Standard (10pts)**	**Meets Standard (15 pts)**	**Above Standard (20 pts)**	**Score**
Individual Performance	There are visuals or table entries missing from at least 5 polygons	There are visuals or table entries missing from at least 3 polygons	There are visuals or table entries missing from at least 1 polygon	There are no missing items for any polygons	
Project Contents	There are 8 or more items missing or not accurately drawn.	There are 5 to 7 items missing or not accurately drawn.	There are 3 to 4 items missing or not accurately drawn.	There are less than 3 items missing or not accurately drawn.	
Creativity, Color, Attention to Details	Lacks neatness, color, very little or no detail.	Somewhat neat, a little color, detail is not representative of the time spent on this project.	Neat, colorful, missing some detail but shows the effort of time spent on the project.	Neat, colorful, creative, visually appealing with great attention to detail.	
Final Presentation	Presentation had many errors. Shows no reasoning or contains reasoning without evidence or incorrect conclusions.	Presentation had a few errors. Shows arguable logic with incorrect conclusion.	Presentation was accurate but with limited details. Shows reasonable logic, conclusion maybe correct but unsupported by data or logic presented.	Presentation had great detail and was accurate. Shows reasonable logic and correct conclusions with support by data and reasoning.	
				Final Score (out of 80)	

Rubric used by students to evaluate group members on participation (average of group member evaluations for each student will be recorded in class participation grade for current unit):

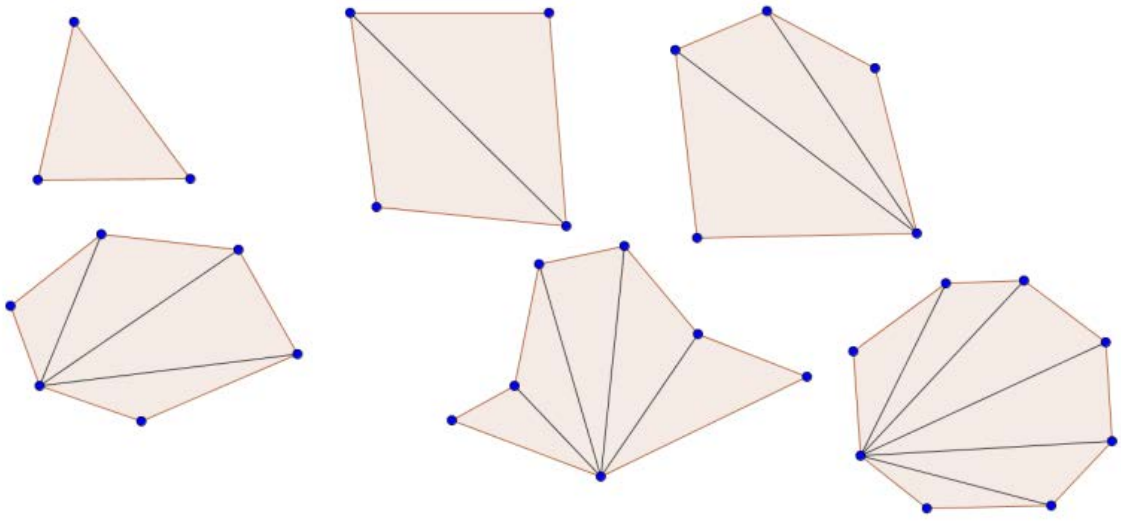
Group Member:	All the time (3)	Some of the time (2)	Never (1)
Participated in group work			
Maintained focus on the task at hand			
Offered help to others, or sought help when needed			

Asked questions that moved the discussion along			
Contributed ideas, opinions, and feelings			
Provided positive feedback to other group members			
	Total Score =		_____/18

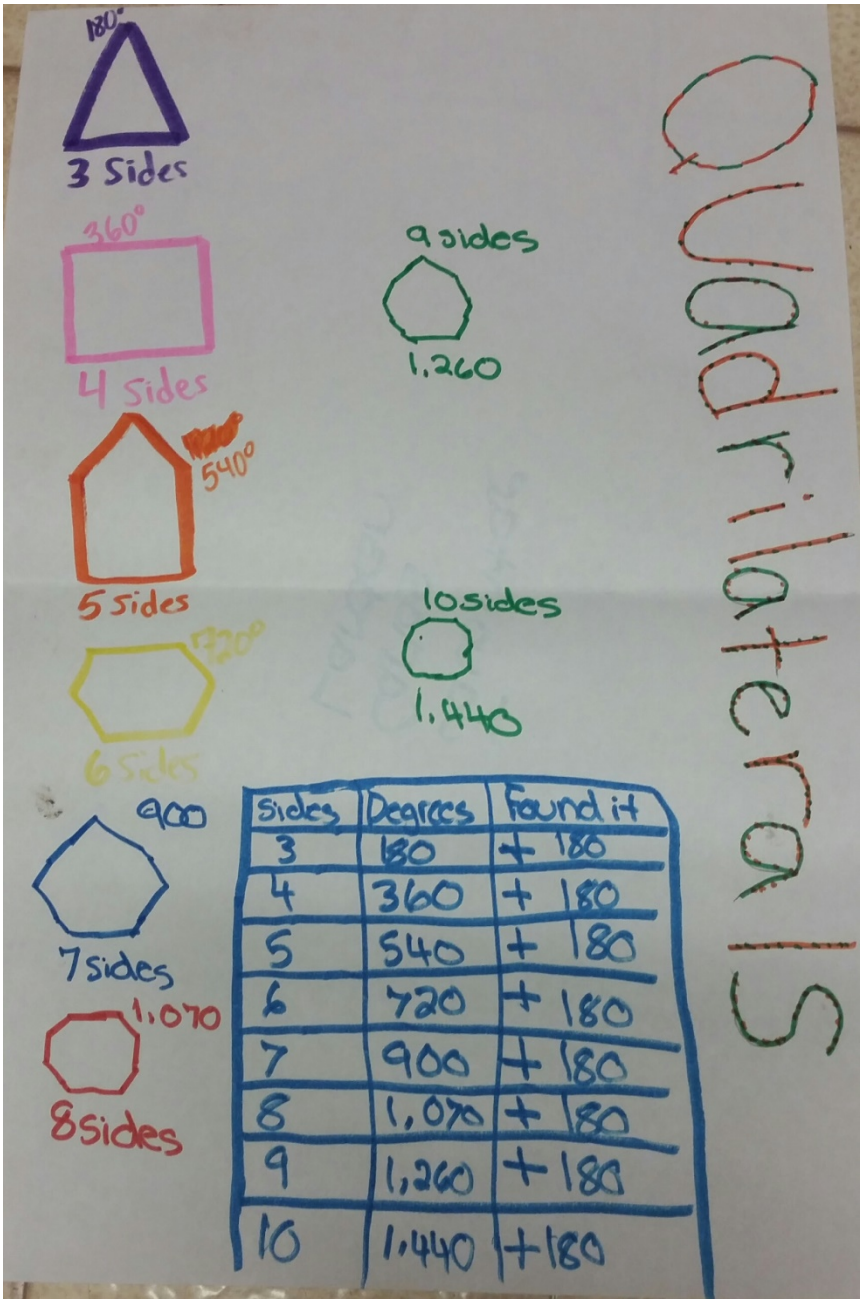
- Examples of student work
  - Student use of craft supplies to start investigation of sum of interior angles:



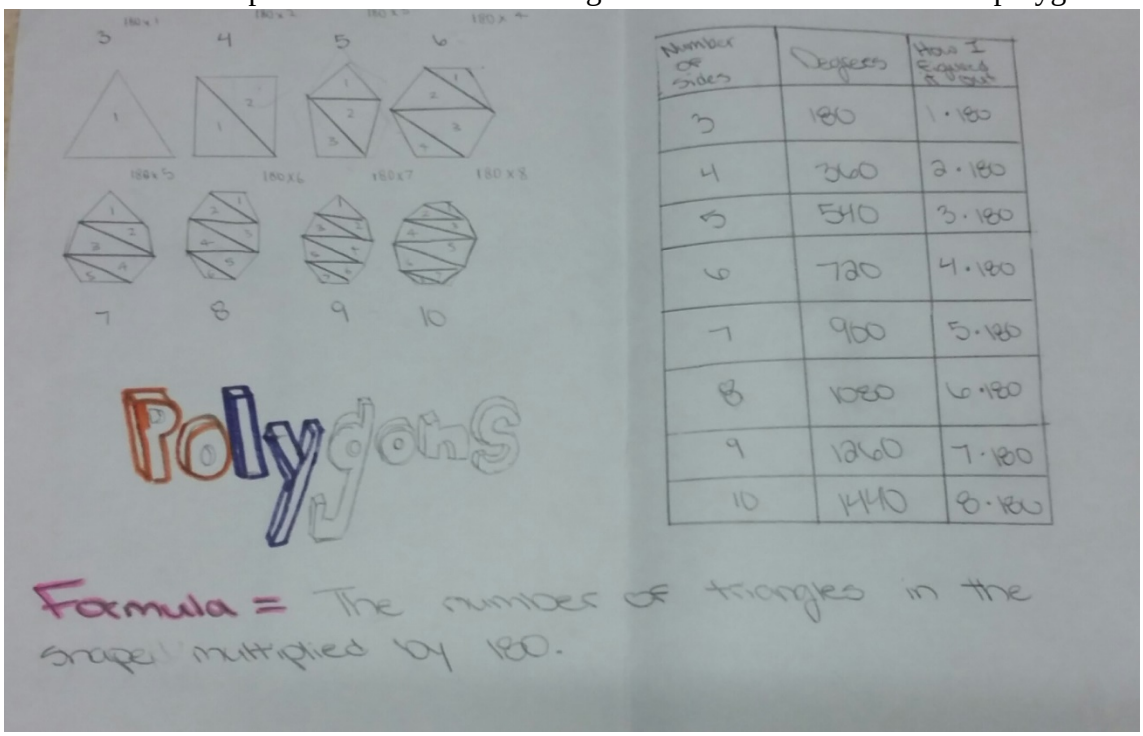
  - Student use of computer software (GeoGebra) to create investigative sketches



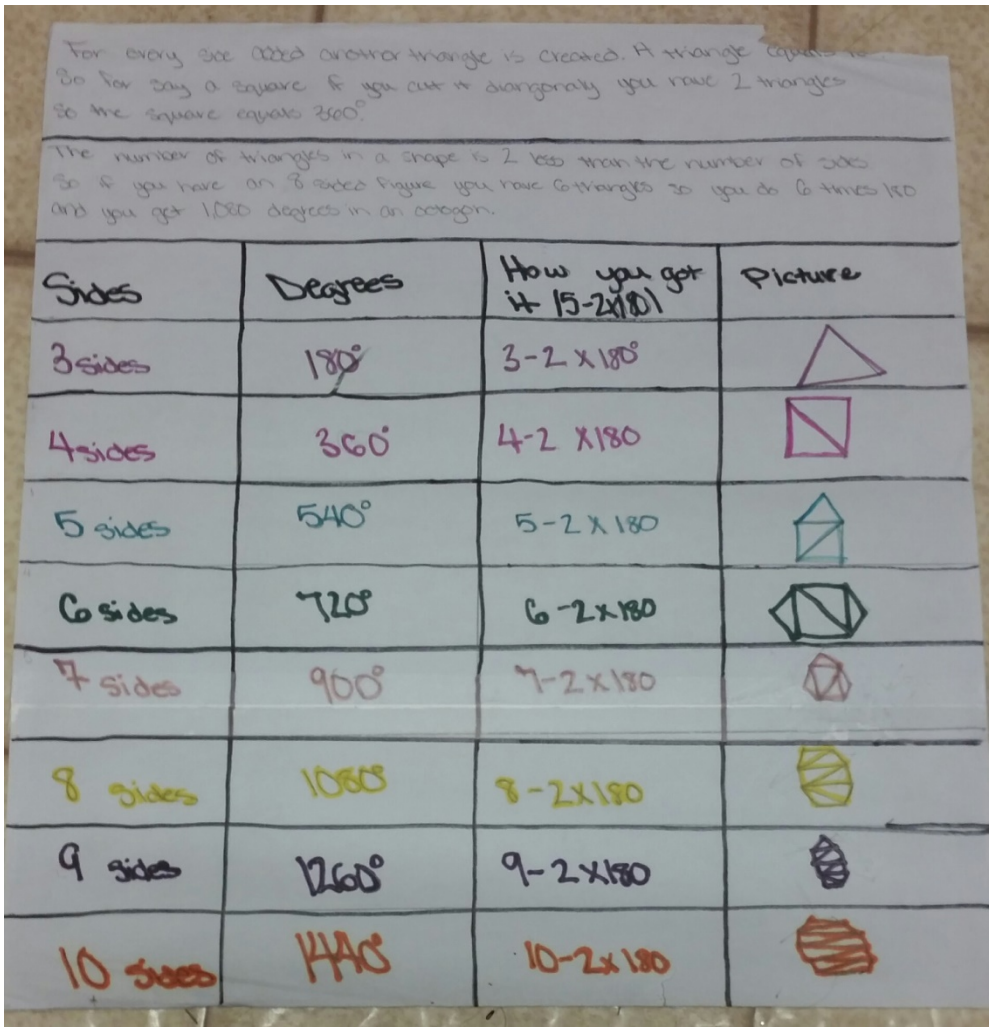
- Incomplete: Incorrect title, Lacks formula or explanation of reasoning



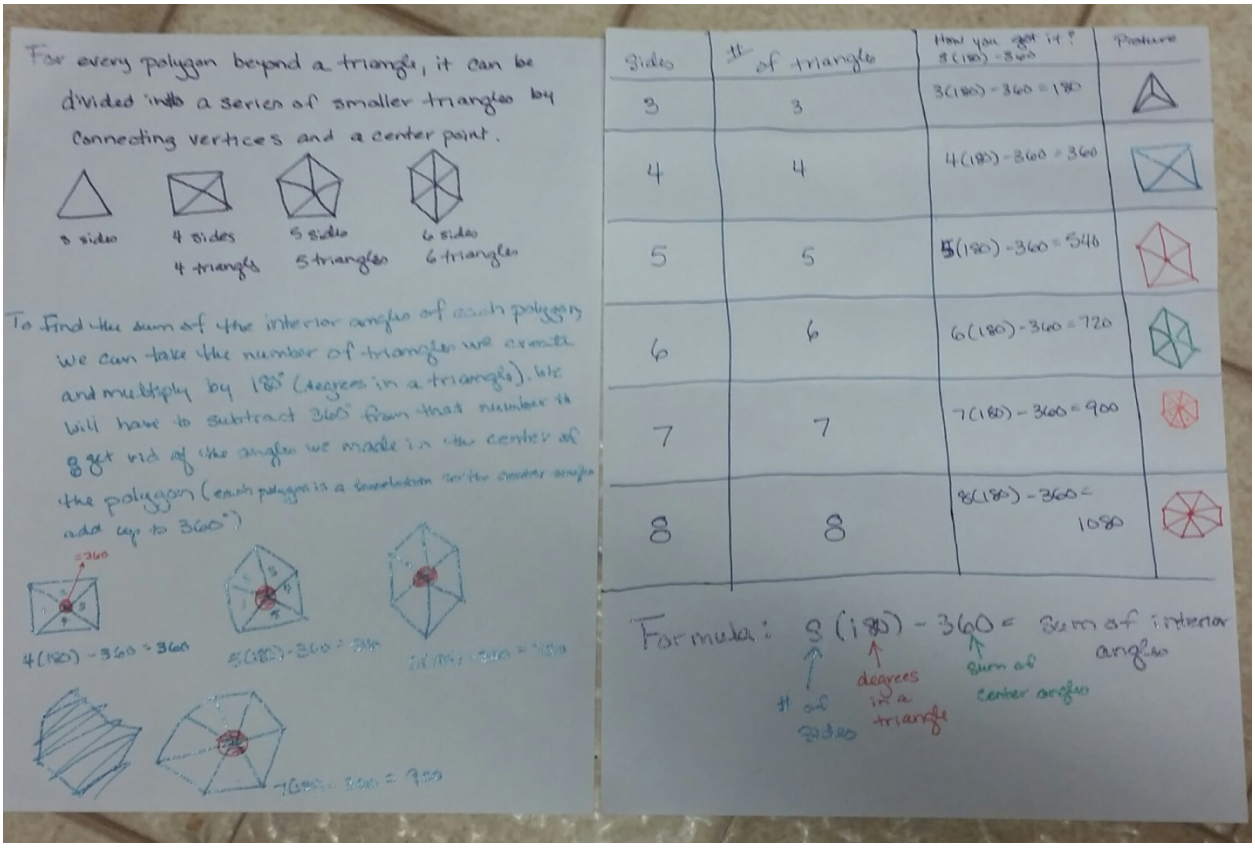
- Developing: Lacks formula, shows understanding of but does not expressly state relationship between number of triangles and number of sides of the polygon



- Complete: Has complete table and models, shows understanding of relationship between number of sides of a polygon and number of triangle in a polygon, explains how answers are reached given a specific number of sides, provides usable method/formula



- Alternate solution: not all students will take the same path to their formulas. This is another example of a complete solution with a different method.



- Click the included like to see an example of an electronic presentation: http://prezi.com/vots-ur0aqix/?utm_campaign=share&utm_medium=copy

Student handout:

Opening Investigation – How much is in there?

A classmate has been absent through the polygons unit and you have volunteered to tutor your classmate to prepare them for the unit assessment. Your classmate has informed you that he learns better with hands-on activities. Using materials you find in your craft closet, you are to model how you and your classmate would create a hands-on method for finding the sum of the interior angles of any polygon and develop a mathematical formula that results from your investigation. Your presentation must include your models, a table of the data you gathered, and your method for find the sum of the interior angles of any polygons

Consider:

1. For each polygon, explain how you know you have found the actual degrees inside that polygon.
2. Expand your method to a polygon with a large number of sides. Does it work? Can you expand your method to a polygon with an unknown number of sides?
3. How would you explain your method to another group or to someone who will have this class next year?

Extension:

1. Can you find the sum of the exterior angles of any polygon? Justify your conclusion.
2. How can we use this information to help design a tile floor for a room in your home?

Polygon Investigation Rubric

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